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Vibration-Test Force Limits Derived from Frequency-Shift Method

Terry D. Scharton*
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California 91109

An improved vibration testing technique has been recently developed and applied to eliminate overtesting caused by the infinite mechanical impedance of the shaker in conventional vibration tests. With the new technique, the acceleration input is notched at the resonance frequencies of the test item in order to limit the shaker force to that predicted for flight. Since few flight vibratory force data are available, the force limits must be calculated from measurements or analyses of the flight mounting structure and test-item mechanical impedances. An improved method of calculating these force limits is derived from evaluating the test-item apparent mass at the coupled-system resonance frequencies. Application of the method to a simple and to a complex coupled oscillator system yields nondimensional results which may be used to calculate limits for future force-limited vibration tests. For example, using the simple system results with Q = 50 and equal impedances of the flight mounting structure and test item, the input acceleration amplitude will be notched by a factor of 31.25 relative to a conventional test.

Nomenclature

$\mathbf{A}$	= interface acceleration
$\mathbf{A}_b$	= base acceleration
$\mathbf{A}_0$	= free acceleration of source
$\mathbf{A}_s$	= acceleration specification
C	= dashpot constant
$\mathbf{F}$	= interface force
$\mathbf{F}_e$	= external force acting on source
$\mathbf{F}_0$	= blocked force of source
$\mathbf{F}_s$	= force specification
k	= spring stiffness
$\mathbf{M}$	= apparent mass, $\mathbf{F}/\mathbf{A}$
M	= residual mass
m	= modal mass
Q	= dynamic amplification factor, $(km)^{0.5}/C$
S_{AA}	= acceleration spectral density
S_{FF}	= force spectral density
α	= ratio of modal to residual mass, m/M
β	= normalized resonance frequency of coupled systems, ω/ω_0
μ	= ratio of load to source residual masses, M_2/M_1
Ω	= ratio of load to source uncoupled natural frequencies, ω_2/ω_1
ω	= radian frequency
ω_0	= natural frequency of uncoupled oscillator, $(k/m)^{0.5}$

Subscripts

1	= source oscillator
2	= load oscillator

Introduction

FOR lightweight aerospace structures, the mechanical impedance of payloads and of the mounting structure are typically comparable, so that the vibration of the combined structure and load involves modest interface forces and responses. Most of the high-amplification resonances and mechanical failures in conventional vibration tests are test artifacts associated with the essentially infinite mechanical impedance and unlimited force capability of the shaker.

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*Member of Technical Staff, Mail Stop 301-456, 4800 Oak Grove Drive. Member AIAA.

With a recently developed vibration testing technique,[1–7] these artificial failures and the related overdesign penalties are eliminated by limiting the vibratory force in the test to that predicted for flight. (Appendix A of the phase I report of Ref. 1 contains 64 background references on the vibration overtest problem.) Limiting the input force is in theory equivalent to limiting the load response, but force limiting is less dependent on the details of the analytical models and is usually more convenient. Also, critical response locations are often numerous and not accessible.

Implementation of force-limited vibration testing requires derivation of a force specification (analogous to that for acceleration), vibration-test fixturing to accommodate force sensors, and shaker operation with dual control of both acceleration and force. This paper focuses on the derivation of force specifications. Fixturing and control are discussed in papers describing applications of force-limited vibration testing at the Jet Propulsion Laboratory (JPL) during the past three years.[8–10]

The following equation, which may be derived[2] from Norton's and Thevenin's equivalent-electrical-circuit theorems, provides a theoretical basis for dual control of vibration tests:

$$1 = \mathbf{A}/\mathbf{A}_0 + \mathbf{F}/\mathbf{F}_0 \qquad (1)$$

Equation (1) is exact, but difficult to apply because the terms on the right-hand side are complex and complicated functions of frequency. (Throughout the paper, boldface type indicates a complex quantity.) The phases of the inputs and the impedances would be difficult to determine, and also phase cannot be specified with currently available vibration-test controllers.

An alternative, approximate formulation for the control of vibration tests is provided by the following extremal equations[2,3,11]:

$$\frac{|\mathbf{A}|}{|\mathbf{A}_s|} \le 1 \quad \text{and} \quad \frac{|\mathbf{F}|}{|\mathbf{F}_s|} \le 1 \qquad (2)$$

In Eq. (2), the free acceleration and blocked force of Eq. (1) are replaced by specifications that envelope the interface acceleration and force in the coupled system.[5–7] With extremal control, the shaker current is automatically adjusted in each narrow frequency band so that the larger of the two ratios in Eq. (2) is equal to unity. At frequencies other than the test-item resonances, the acceleration specification usually controls the test level; at the resonances, the base reaction force increases and the force specification limits the input.

Most vibration controllers have the capability for extremal control, but older controllers allow only one reference specification. To implement dual control in this case, a filter must be used to scale the shaker force feedback signal to an equivalent acceleration.[4] New controllers allow separate specifications for limit channels, so Eq. (2)

may be directly implemented. Force limiting has been used primarily for random-vibration tests, but the application to swept-sine tests is also practical and beneficial.

Frequency-Shift Method

There are virtually no flight data and few system test data on the vibratory forces at mounting-structure–test-item interfaces. Currently force limits for vibration tests are therefore calculated using analytical or measured structural impedances of the mounting structure and the test items, together with the conventional interface acceleration specification. Herein an improved method of calculating the force limits is described and applied to a simple and to a complex two-degree-of-freedom system (TDFS). Some of the rationale for the method was provided by Smallwood in Ref. 6.

The frequency-shift method[12,13] will first be described with reference to the simple TDFS in Fig. 1. The two oscillators in Fig. 1 represent coupled resonant modes of the source and load.

For both the flight configuration with a coupled source and load and the vibration-test configuration with an isolated load, the interface force spectral density is related to the interface acceleration spectral density as

$$S_{FF}(\omega) = |\mathbf{M}_2(\omega)|^2 S_{AA}(\omega) \qquad (3)$$

The load apparent mass (the reciprocal of accelerance) is a frequency response function (FRF) that includes mass, damping, and stiffness effects. The frequency dependence is shown explicitly in Eq. (3) to emphasize that the relation between force and acceleration applies at each frequency.

It can be shown[12] that, for white-noise base motion or external force excitation of the coupled system in Fig. 1, the interface acceleration and force spectral densities both peak at the same frequencies, i.e, the coupled system natural frequencies. The load apparent mass, evaluated at one of these natural frequencies, may be interpreted as the ratio of the force spectral peak to the acceleration spectral peak at that natural frequency.

The frequency-shift method of deriving force specifications consists in multiplying the conventional acceleration specification, which is assumed to properly envelope the acceleration spectral peaks, by the load apparent mass, evaluated at a coupled system resonance frequency. A central point of the method is that the load apparent mass must be evaluated at the coupled system, or shifted, resonance frequencies. The values of the load apparent mass at the coupled-system resonance frequencies are considerably less than the peak value at the load uncoupled resonance frequency.

Figure 2 illustrates the application of Eq. (3) to the simple TDFS model shown in Fig. 1 for the special case of identical oscillators, unit masses, unit base acceleration, and a Q of 50. The ordinates in Fig. 2 are FRF magnitudes, and for convenience the results are discussed in terms of a sinusoidal input. The abscissa in Fig. 2 is frequency, normalized by the resonance frequency of an uncoupled oscillator. The top curve in Fig. 2 is the magnitude of the coupled-system interface force, which is the product of the load apparent mass and the interface acceleration as indicated in Eq. (3). The middle curve in Fig. 2 is the magnitude of the coupled-system interface acceleration. The bottom curve is the magnitude of the load apparent mass, which peaks at the load uncoupled resonance frequency ω_0 with an amplitude Q.

In Fig. 2, the force peak of 80 is calculated by multiplying the acceleration peak of 50 by the load apparent-mass value of 1.6, at the lower resonance frequency. In a conventional vibration test without force limiting, the corresponding shaker force would be of 2500, which is the acceleration envelope of 50 times 50, the load apparent mass at the load resonance frequency. With force limiting, the input acceleration would be notched at the load resonance frequency to reduce the shaker force by a factor of 2500/80, or 31.25.

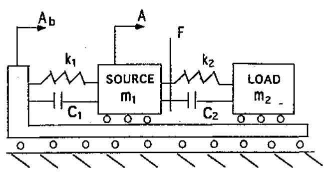


Fig. 1 Simple TDFS of coupled oscillators.

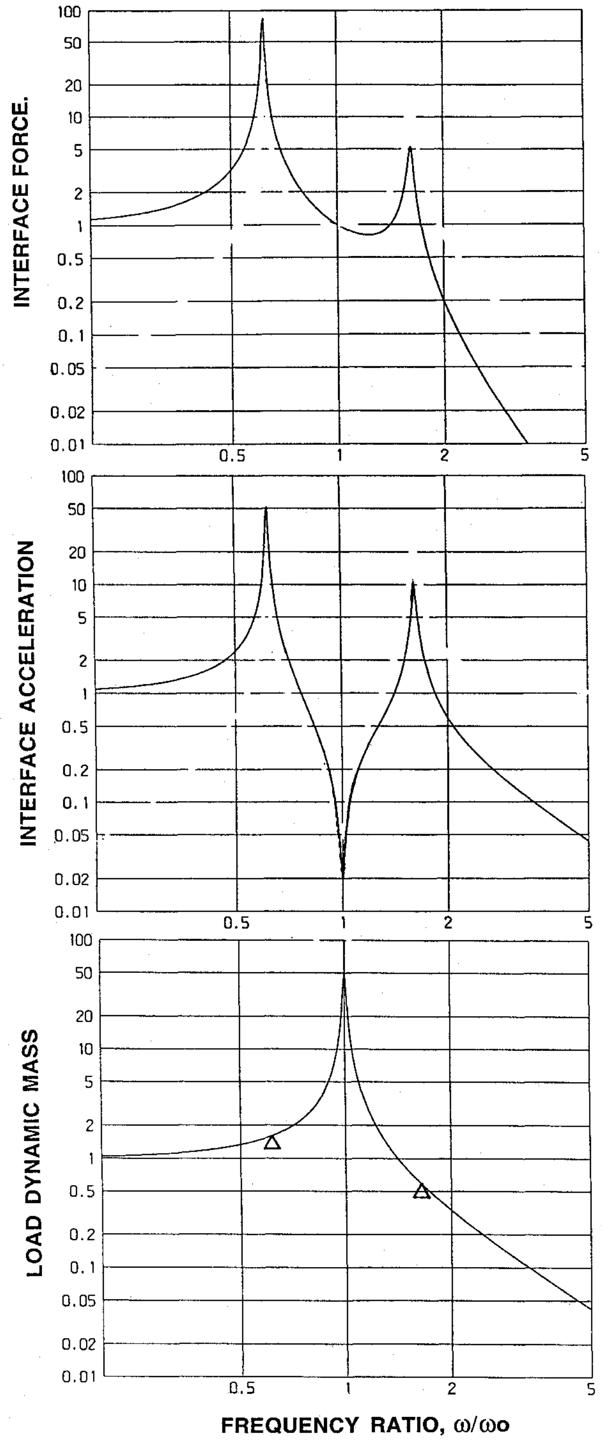


Fig. 2 Force, acceleration, and apparent mass for simple TDFS ($\omega_1 = \omega_2 = \omega_0$, $m_1 = m_2 = 1$, $A_b = 1$, $Q = 50$).

Application to Simple TDFS

As a first example of the frequency-shift method, the force limit is calculated for the TDFS in Fig. 1 with different masses of the source and the load oscillators.[12] For this TDFS, the maximum response of the load and therefore the maximum interface force occur when the uncoupled resonance frequency of the load equals that of the source.[14] For this case, the characteristic equation is that of a classical dynamic absorber[15]:

$$\left(\frac{\omega}{\omega_0}\right)^2 = 1 + \frac{m_2/m_1}{2} \pm \left((m_2/m_1) + \frac{(m_2/m_1)^2}{4}\right)^{0.5} \qquad (4)$$

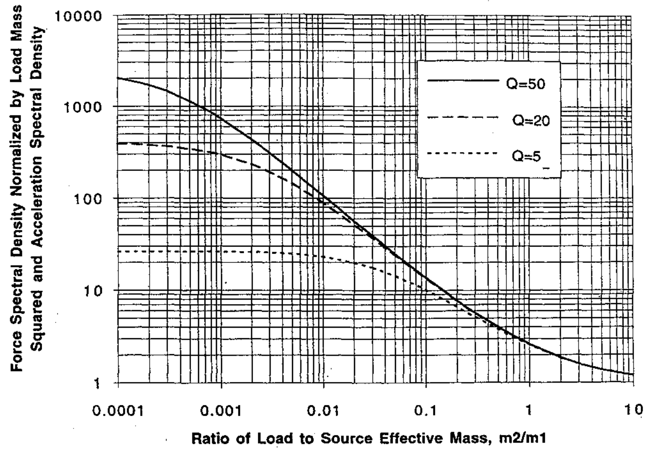


Fig. 3 Maximum normalized force for simple TDFS.

The ratio of the interface force to acceleration spectral densities, calculated as in Eq. (3) from the magnitude squared of the load apparent mass, is

$$\frac{S_{FF}}{S_{AA}m_2^2} = \frac{1 + (\omega/\omega_0)^2/Q_2^2}{\left[1 - (\omega/\omega_0)^2\right]^2 + (\omega/\omega_0)^2/Q_2^2} \tag{5}$$

The force spectral density, normalized by the load mass squared and by the acceleration spectral density, at the two coupled system resonances is obtained by combining Eqs. (4) and (5). For this TDFS the normalized force is just slightly larger at the lower resonance frequency of Eq. (4). The maximum normalized-force spectral density, obtained by evaluating Eq. 5 at the lower resonance frequency from Eq. 4, is plotted against the ratio of load to source mass for three values of Q_2 in Fig. 3.

In Fig. 3, for very small (0.0001) values of the ratio of load to source mass, the load has little effect on the source, and the maximum normalized force approaches Q squared. For larger ratios of the masses, the maximum force is smaller because of the vibration absorber effect at the load resonance frequency. For equal load and source masses, the maximum normalized force in Fig. 3 is 2.56, or $(1.6)^2$, as in the numerical example of Fig. 2.

Use of Fig. 3 to define force specifications requires that the oscillator masses in Fig. 1 be interpreted as frequency dependent masses of the distributed source and load system. It has proven convenient to define the masses in one-third-octave frequency bands. In most previous force-limited vibration tests[8–10] the masses have been taken as the smoothed FRFs of the ratio of drive-point force to acceleration as measured with a shaker or an impact hammer. This smoothing is defined by geometric averaging in the frequency domain, and the result is synonymous with what is sometimes called the critically damped, asymptotic, or skeleton FRF. Alternatively, using the results of finite-element-model (FEM) analyses, the masses in Fig. 3 have been taken as the suitably normalized[16] residual masses, i.e., the sum of the masses of all modes with resonance frequencies above the excitation frequency band.

Defining the masses in Fig. 3 as smoothed FRFs or as residual masses, instead of as modal masses, overestimates both the load and source masses, which in turn overestimates the interface force calculated from Fig. 3. This approach was thought to be conservative for testing, but conceptually it is not very satisfying. The Fig. 1 model, with a single mass for the source and the load, is deficient in that it cannot represent the force contributions of both resonant and nonresonant structural modes. This leads to the consideration of a more complex model.

Application to Modal- and Residual-Mass TDFS

As a second example of the frequency-shift method, the force limit is calculated for a more complex TDFS model in which the source and load each have two masses to represent both the residual and modal masses of a continuous system.[12] It is assumed that the acceleration specification correctly envelopes the higher of the two acceleration peaks of the complex TDFS system. For this system, the calculation of the force limit requires taking into account the relative sizes of the acceleration peaks at the two coupled-system resonance frequencies, which requires some specific assumptions about how the system is excited. Calculation of the force limit for this system also necessitates a tuning analysis, in which the maximum force is calculated for different ratios of the load and source uncoupled resonance frequencies. The complexity of the model requires that the results be presented in tabular form for different ratios of modal to residual mass, for both the source and the load.

Figure 4a shows a model of a source or a load in which each mode may be represented as a single-degree-of-freedom system attached to a common interface. (This is sometimes called an asparagus-patch model.) Derivation of this type of model from an analysis requires normalizing the modes so that the interface reaction forces equal the modal inertial forces.[16] With this type of normalization, the modal masses become the physical masses of oscillators attached to the interface, and are called effective modal masses.

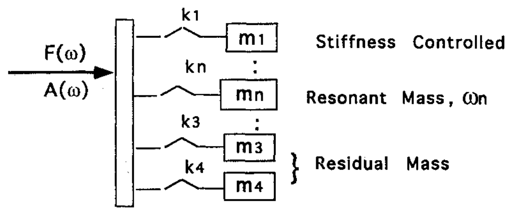
When this model is excited at the interface at a frequency near the resonance frequency ω_n of the nth mode, the model may be simplified to that in Fig. 4b, where m_n is the modal mass of the nth mode and M_n is the residual mass, i.e., the sum of the masses of all higher-resonance-frequency modes. Finally, Fig. 4c shows a coupled system with a residual- and modal-mass model of both source and load. The ratio of modal to residual mass is $\alpha_1 = m_1/M_1$ for the source and $\alpha_2 = m_2/M_2$ for the load; the ratio of load and source uncoupled resonance frequencies is $\Omega = \omega_2/\omega_1$; and the ratio of load and source residual masses is $\mu = M_2/M_1$.

The undamped resonance frequencies of the coupled system in Fig. 4c are solutions of

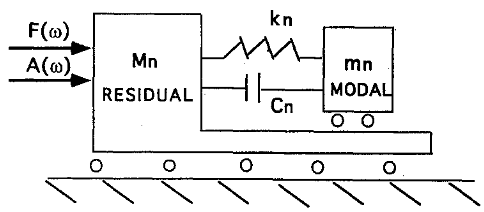
$$\left(1-\beta_1^2\right)\left(1-\beta_2^2\right) + \alpha_1\left(1-\beta_2^2\right) + \mu\left(1-\beta_1^2\right)\left(1-\beta_2^2\right) + \mu\alpha_2\left(1-\beta_1^2\right) = 0 \tag{6}$$

with $\beta_1 = \omega/\omega_1$, $\beta_2 = \omega/\omega_2$, $\omega_1 = (k_1/m_1)^{0.5}$, and $\omega_2 = (k_2/m_2)^{0.5}$. Using Ω to eliminate β_1, the two undamped resonance frequencies of the coupled system are found from the quadratic formula:

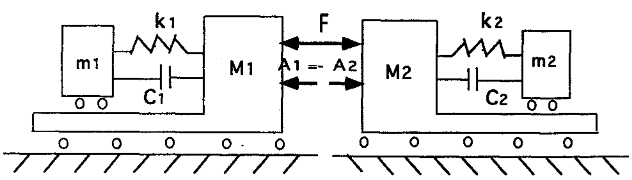
$$\beta_2^2 = -B/2 \pm (B^2 - 4C)^{0.5}/2 \tag{7}$$



a) Asparagus patch model



b) Residual and modal mass model



c) Coupled residual and modal mass models

Fig. 4 Complex TDFS with residual and modal masses.

Table 1 Maximum force spectral density, for $Q = 20$, normalized by load residual mass squared and acceleration spectral density

Ratio of modal to residual mass	Residual mass ratio, M2/M1								
m1/M1, m2/M2	0.001	0.003	0.01	0.03	0.1	0.3	1	3	10
8.0, 8.0	932	933	936	948	1001	1180	1240	1234	1238
8.0, 4.0	233	233	233	235	239	256	294	265	250
8.0, 2.0	58	58	58	58	59	60	68	73	68
8.0, 1.0	15	15	15	15	15	15	17	23	22
8.0, 0.5	4	4	4	4	4	4	4	7	6
8.0, 0.25	1	1	1	1	1	1	1	2	5
8.0, 0.125	1	1	1	1	1	1	1	1	3
4.0, 8.0	871	867	858	849	904	1042	1067	1110	1229
4.0, 4.0	218	218	217	216	220	250	254	250	252
4.0, 2.0	55	55	55	55	56	61	72	68	67
4.0, 1.0	14	14	14	14	14	16	21	23	22
4.0, 0.5	3	3	4	4	4	4	6	10	10
4.0, 0.25	1	1	1	1	1	1	2	5	5
4.0, 0.125	1	1	1	1	1	1	1	3	3
2.0, 8.0	1586	1478	1260	1061	990	946	982	1099	1201
2.0, 4.0	406	391	355	305	272	259	238	236	254
2.0, 2.0	103	101	97	88	79	82	70	65	62
2.0, 1.0	26	26	26	25	24	25	25	23	22
2.0, 0.5	7	7	7	7	7	9	10	10	10
2.0, 0.25	2	2	2	2	2	3	5	5	6
2.0, 0.125	1	1	1	1	1	1	3	3	4
1.0, 8.0	11041	5731	2714	1486	967	901	984	1095	1181
1.0, 4.0	3869	2206	1105	567	332	247	233	238	248
1.0, 2.0	1228	826	432	226	125	83	71	66	64
1.0, 1.0	359	283	166	100	50	34	26	23	23
1.0, 0.5	100	89	63	42	24	15	12	11	11
1.0, 0.25	28	27	23	17	11	8	6	6	6
1.0, 0.125	8	8	8	7	5	5	4	4	4
0.5, 8.0	13889	7720	3501	1726	1023	880	974	1093	1171
0.5, 4.0	4516	2895	1417	695	357	247	225	240	244
0.5, 2.0	1346	1003	561	283	136	89	70	64	65
0.5, 1.0	377	319	211	117	59	39	27	24	22
0.5, 0.5	102	95	74	48	27	17	12	11	10
0.5, 0.25	28	27	25	19	13	8	7	6	6
0.5, 0.125	8	8	8	8	6	5	4	4	4
0.25, 8.0	17378	9978	4092	1944	1017	833	936	1092	1166
0.25, 4.0	5194	3725	1805	812	380	249	225	241	242
0.25, 2.0	1455	1205	741	359	173	93	71	66	65
0.25, 1.0	391	354	269	160	74	43	28	23	22
0.25, 0.5	103	99	86	63	38	22	14	12	11
0.25, 0.25	28	28	27	23	16	10	8	7	7
0.25, 0.125	8	8	8	8	7	5	5	4	4

where

$$B = \frac{-(1+\mu+\alpha_1)/\Omega^2 - (1+\mu+\mu\alpha_2)}{1+\mu}$$

and

$$C = \frac{1+\mu+\alpha_1+\mu\alpha_2}{(1+\mu)\Omega^2}$$

The interface force spectral density, normalized by the acceleration spectral density and the load residual mass squared, calculated as in Eq. (3) from the magnitude squared of the load apparent mass, is

$$\frac{|\mathbf{M}_2|^2}{\mathrm{M}_2^2} = \frac{\left[\left(1-\beta_2^2\right)+\alpha_2\right]^2 + \beta_2^2(1+\alpha_2)^2/Q_2^2}{\left(1-\beta_2^2\right)^2 + \beta_2^2/Q_2^2} \tag{8}$$

Combining Eqs. (7) and (8) yields the normalized-force spectral density at each of the two coupled-system resonance frequencies.

The desired result is the ratio of the larger of the two force spectral-density peaks to the larger of the two acceleration spectral-density peaks, the former being the desired force limit and the latter corresponding to the acceleration specification. Unfortunately, the peak acceleration and peak force do not necessarily occur at the same frequency, e.g., the peak acceleration may be at the higher of the two coupled-system resonance frequencies while the peak force is at the lower of the two frequencies. This in fact occurs when the uncoupled resonance frequencies of the load and source are approximately equal, that is, for Ω near unity.

To obtain the desired result, it is necessary to calculate the ratio of the two acceleration spectral-density peaks of the coupled system, and this ratio depends on how the system is excited. Herein, it is assumed that the modal mass of the source is excited by an external force with a flat spectral density over the frequency band including the two resonance frequencies of the coupled system. (Two other excitation possibilities would be that the spectrum of the free acceleration or of the blocked force of the residual mass of the source is independent of frequency. The flat external force acting on the source modal mass is chosen because it is thought to be the most typical and because it yields the highest force limits when the load and source masses are comparable.)

For any excitation of the source system, the magnitude squared of the ratio of the interface acceleration $\mathbf{A}$ to the free acceleration $\mathbf{A}_{10}$ of the residual mass of the source is

$$\left|\frac{\mathbf{A}}{\mathbf{A}_{10}}\right|^2 = \left|\frac{\mathbf{M}_1}{(\mathbf{M}_1 + \mathbf{M}_2)}\right|^2 \tag{9}$$

where $\mathbf{M}_1$ is obtained from Eq. (8) by replacing the subscript 2 with 1. For the chosen form of excitation, an external force $\mathbf{F}_e$ acting on the source modal mass m_1, the magnitude squared of the free acceleration $\mathbf{A}_{10}$ is

$$\left|\frac{\mathbf{A}_{10}}{\mathbf{F}_e/m_1}\right|^2 = \frac{\beta_1^4\left(1+\beta_1^2/Q_1^2\right)}{\left[\left(1-\beta_1^2\right)\left(1-\beta_1^2/\alpha_1\right)-1\right]^2 + \beta_1^6(1+1/\alpha_1)^2/Q_1^2} \tag{10}$$

The free acceleration is eliminated between Eqs. (9) and (10), and the interface acceleration at the two coupled system resonances is determined using Ω to eliminate β_1 and substituting β_2^2 from Eq. (7). Assuming that the external-force spectrum is the same at the lower and upper resonance frequencies of the coupled system, we obtain the ratio of the interface acceleration spectral density peaks at these two frequencies.

The apparent mass in Eq. (8) is scaled by multiplying the apparent mass at the resonance frequency corresponding to the smaller acceleration peak by the ratio of the smaller to the larger acceleration peak and by multiplying the apparent mass at the other frequency by unity. Finally, the larger of the two thus scaled apparent masses is used as the ratio of the greater force spectral density peak to the greater acceleration spectral density peak.

The final step in the derivation of the force limit is to vary the ratio, $\Omega = \omega_2/\omega_1$, of the uncoupled resonance frequencies of the load to the source to insure that the maximum value of the interface force is found for all mass, stiffness, and damping combinations for the system in Fig. 4c. A tuning analysis is conducted in which the value of the frequency ratio Ω squared is varied in steps of $\frac{1}{16}$ from $\frac{8}{16}$ to $\frac{32}{16}$, which corresponds to 3% increments in the frequency ratio. The maximum tuned values of the force spectral density, normalized by the load residual mass squared and the maximum value of the acceleration spectral density (which is equivalent to the acceleration specification), are listed in Table 1 for the amplification factors Q_1 and Q_2 both equal to 20. [Results for other Q's are available from the author or may be computed from Eqs. (6–10).]

The normalized force spectral density in Table 1 is unity when $\alpha_2 = 0$ (see Eq. 8) and should be interpolated for $0.125 > \alpha_2 > 0$. It is suggested that the force spectral density for $\alpha_1 < 0.25$, be taken as the value at $\alpha_1 = 0.25$. (Small α_1's correspond to local modes of the source, which may not be relevant to the interface environment. Also, the interface force for a source modal mass of zero is different than the asymptotic value for $\alpha_1 = 0$, which corresponds to a very small mass moving at very large amplitude.)

To use the force limits in Table 1, both the residual and the modal masses of the source and load must be known as functions of frequency, either from a test, from an FEM, or from both. FEM analyses with the modal masses normalized as in Ref. 16 provide both the modal and residual effective masses. If shaker or tap tests are used to measure the effective masses, the smoothed FRF of the magnitude of the ratio of force to acceleration should be taken as the effective residual mass. The effective modal mass is the negative change in the effective residual mass at the resonance frequencies.

Concluding Remarks

A new method of calculating force specifications for force-limited vibration tests has been derived and applied to two systems, a simple and a complex TDFS. At JPL, force limits are calculated by enveloping the results presented herein for both the simple TDFS-and the residual- and modal-mass models. FEM calculations of effective mass are used primarily to set force limits for design and for vibration tests conducted early in the programs with development and engineering hardware. The effective mass of the test item is always updated with the results of a low-level sine-sweep test, conducted just before the random-vibration test. Modal-tap test data for the mounting-structure effective mass are almost always obtained before the final vibration test of the flight hardware is conducted.

There is great need for flight measurements of the vibratory forces at equipment–mounting-structure interfaces (for example, at the spacecraft–launch-vehicle interface) to complement the existing acceleration data base and to validate this and other force-limit prediction methods.

Acknowledgments

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E. A. Thornton
Associate Editor